1/3 2 вар 2134 [redacted]

1 $\dfrac{(1+i)^3}{(1-\sqrt{3}\,i)^4} = \dfrac{(\sqrt{2}\,e^{i\frac{\pi}{4}})^3}{(2\,e^{-i\frac{\pi}{6}})^4} = \dfrac{2\sqrt{2}\,e^{i\frac{3\pi}{4}}}{16\,e^{-i\frac{2\pi}{3}}} = \dfrac{\sqrt{2}}{8}\,e^{i\left(\frac{9\pi}{12}+\frac{8\pi}{12}\right)}$

$1+i = \sqrt{2}\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right) = \sqrt{2}\,e^{i\frac{\pi}{4}}$

$1-\sqrt{3}\,i = 2\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right) = 2\,e^{-i\frac{\pi}{6}}$

ответ: $\boxed{|z| = \frac{\sqrt{2}}{8} \quad \arg z = -\frac{\pi}{12}}$ да

1	2	3	4	5	6	7	8	9	10	11	12
+	+	+	+	+	+	+	+	+	+	+	−

$\Sigma = 11$ баллов

5−

2. $f(z) = x^2 - y^2 - i\,2xy$

$u(x,y) = x^2 - y^2 \quad u'_x = 2x \quad u'_y = -2y$

$v(x,y) = -2xy \quad v_x = -2x \quad v'_y = -2y$

$\begin{cases} u'_x = v'_y \\ u'_y = -v'_x \end{cases} \quad \begin{matrix} 2x = -2x \\ -2y = +2y \end{matrix} \Rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases}$

ответ: дифф-ма в т. (0,0) да

3 $f(z) = 2(\operatorname{ch} z - 1) - z^2 = 2\left(\dfrac{e^z + e^{-z}}{2} - 1\right) - z^2 =$

~~$f(z) = 2(\operatorname{ch} z - 1) - z^2$~~

$= \sum\limits_{n=0}^{\infty} \dfrac{z^n}{n!} + \sum\limits_{n=0}^{\infty} \dfrac{(-1)^n z^n}{n!} - 2 - z^2 = 1 + z + \dfrac{z^2}{2} + \dfrac{z^3}{6} +$

$+ \sum\limits_{n=4}^{\infty} \dfrac{z^n}{n!} + 1 - z + \dfrac{z^2}{2} - \dfrac{z^3}{6} + \sum\limits_{n=4}^{\infty} (-1)^n \dfrac{z^n}{n!} - 2 - z^2 =$

$= \sum\limits_{n=4}^{\infty} \dfrac{z^n}{n!} + \sum\limits_{n=4}^{\infty} (-1)^n \dfrac{z^n}{n!} = \dfrac{z^4}{4!} \cdot 2 + \sum\limits_{n=5}^{\infty} \dfrac{z^n}{n!} + \sum\limits_{n=5}^{\infty} (-1)^n \dfrac{z^n}{n!}$

$= z^4 \left(\dfrac{1}{12} + \sum\limits_{n=5}^{\infty} \dfrac{z^{n-5}}{n!} + \sum\limits_{n=5}^{\infty} (-1)^n \dfrac{z^{n-5}}{n!}\right) = z^4 g(z)$

$g(0) = \frac{1}{12} \neq 0$ верно

ответ → нуль порядка 4

4) $f(z) = \frac{z+2}{z^2+2z-8}$

$2<|z|<4$

$f(z) = \frac{z+2}{z^2+2z-8} = \frac{z+2}{(z+4)(z-2)}$

решение см на 3 листе

5) $z^3 e^{\frac{1}{z}} = z^3 \sum_{n=0}^{\infty} \frac{1}{n!} \frac{1}{z^n} = \sum_{n=0}^{\infty} \frac{1}{n!} z^{-n+3} \underset{k=n-3}{=} \sum_{n=-3}^{\infty} \frac{1}{(n+3)!} z^{-n}$

$= \sum_{n=-3}^{0} \frac{1}{(n+3)!} z^{-n} + \sum_{n=1}^{\infty} \frac{1}{(n+3)!} z^{-n} = z^3 + z^2 + \frac{z}{2} + \frac{1}{6} +$

$+ \sum_{n=1}^{\infty} \frac{1}{(n+3)! z^{-n}}$ ← ответ

да

6) $f(z) = \frac{z}{(z+1)^2(z-2)}$

$z_1 = 2$ - полюс 1 пор-ка

$z_2 = -1$ полюс 2 пор-ка

$\underset{z=2}{\operatorname{res}} f(z) = \lim_{z\to 2} \frac{z}{(z+1)^2} = \frac{2}{9}$ да

да

$\underset{z=-1}{\operatorname{res}} f(z) = \lim_{z\to -1} \frac{d}{dz}\left(\frac{z}{(z-2)^2}\right) = \frac{z-2-z}{(z-2)^2} = -\frac{2}{9}$

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7) $\oint\limits_{|z|=2} \frac{ch(iz)}{(z^2+1)^2}\,dz$ $\quad z^2=-1$

$z_{1,2}=\pm i$ — полюсы 2-го пор. попадают внутр контура

$$\operatorname*{res}_{z=i} f(z) = \lim_{z\to i}\frac{d}{dz}\left((z-i)^2\frac{ch(iz)}{(z-i)^2(z+i)^2}\right) = \cancel{\lim}$$

$$= \lim_{z\to i}\left(\frac{e^{iz}}{2(z+i)^2}+\frac{e^{-iz}}{2(z+i)^2}\right) = \lim_{z\to i}\left(\frac{ie^{iz}(z+i)^2-2(z+i)e^{iz}}{2(z+i)^4}+\right.$$

$$\left.+\frac{-ie^{-iz}(z+i)^2-2(z+i)e^{-iz}}{2(z+i)^4}\right) = \lim_{z\to i}\left(\frac{ie^{iz}}{2(z+i)^2}-\frac{e^{iz}}{(z+i)^3}\right.$$

$$\left.-\frac{ie^{-iz}}{2(z+i)^2}-\frac{e^{-iz}}{(z+i)^3}\right) = \frac{ie^{-1}}{2(2i)^2}-\frac{e^{-1}}{(2i)^3}-\frac{ie}{\underset{-8}{2(2i)^2}}-\frac{e}{(2i)^3}=$$

$$=\frac{i}{-8e}-\frac{i}{8e}+\frac{ie}{8}-\frac{ei}{8}=-\frac{i}{4e}$$

$$\operatorname*{res}_{z=-i} f(z) = \lim_{z\to -i}\left(\frac{ch(iz)}{(z-i)^2}\right)' = \lim_{z\to i}\left(\frac{e^{iz}}{2(z-i)^2}+\frac{e^{-iz}}{2(z-i)^2}\right)'$$

$$=\lim_{z\to -i}\left(\frac{ie^{iz}}{2(z-i)^2}-\frac{e^{iz}}{(z-i)^3}-\frac{ie^{-iz}}{2(z-i)^2}-\frac{e^{-iz}}{(z-i)^3}\right)=$$

$$=\frac{ie}{2(-2i)^2}-\frac{e}{(-2i)^3}-\frac{ie^{-1}}{2(-2i)^2}-\frac{e^{-1}}{(-2i)^3}=\frac{ie}{-8}-\frac{e}{8i}+\frac{i}{8e}-\frac{1}{8ie}-$$

$$=\frac{-ie}{8}+\frac{ei}{8}+\frac{i}{8e}+\frac{i}{8e}=\frac{i}{4e}$$

$$I=2\pi i\left(-\frac{i}{4e}+\frac{i}{4e}\right)=0$$ ← ответ

да

8) $\int_0^{2\pi} \frac{dx}{3+\cos x} = \left| \begin{matrix} z=e^{ix} \quad dz = i\,dx\,z \quad dx = \frac{dz}{iz} \\ \cos x = \frac{e^{ix}+e^{-ix}}{2} = \frac{z+\frac{1}{z}}{2} = \frac{z^2+1}{2z} \end{matrix} \right| = \frac{1}{i} \oint_{|z|=1} \frac{dz}{z\left(3+\frac{z^2+1}{2z}\right)}$

$\frac{2}{z\left(6+z+\frac{1}{z}\right)}$

$6+z^2+1$

$= \frac{2}{i} \oint \frac{dz}{z(6+z^2+1)}$

$I = 2\pi i \operatorname{res}_{z_1} \left(\frac{2}{(z^2+6z+1)} \right) = 2\pi \frac{2}{2z+6}\Big|_{z=z_1} \ominus$

$D = 36-4 = 32$

$z_{1,2} = \frac{-6 \pm \sqrt{32}}{2}$

$= \frac{-6 \pm 4\sqrt{2}}{2} = -3 \pm 2\sqrt{2}$

$z_1 = -3+2\sqrt{2}$

внутри контура

$\ominus \; 2\pi \frac{1}{z+3}\Big|_{z_1} = 2\pi \frac{1}{2\sqrt{2}} = \frac{\pi\sqrt{2}}{2}$ верно

Ответ

9) $\int_{-\infty}^{+\infty} \frac{dx}{(x^2+2x+2)^2} = \left| \begin{matrix} f(z) = \frac{1}{(z^2+2z+2)^2} \\ D = 4-8 = -4 \\ z_{1,2} = -1 \pm i \\ \text{полюсы 2 пор-а} \\ z_1 = -1+i \end{matrix} \right| =$

$= 2\pi i \operatorname{res}_{z_1} f(z) = 2\pi i \lim_{z \to z_1} \frac{d}{dz} \left(\frac{1}{(z+1+i)^2} \right) =$

$= -4\pi i \frac{1}{(-1+i+1+i)^3} = \frac{-4\pi i}{-8i} = \frac{\pi}{2}$ да

10) $\int_0^{+\infty} \frac{x \sin 5x \, dx}{(x^2+16)(x^2+36)} = \frac{1}{2} \int_{-\infty}^{+\infty} \frac{-ix e^{i5x} \, dx}{(x^2+16)(x^2+36)} = 2\pi i \left(-\frac{1}{2}\right) i$

$\cdot \operatorname{res}_{4i} \frac{z e^{i5z}}{(z^2+16)(z^2+36)} + \operatorname{res}_{6i} \frac{z e^{i5z}}{(z^2+16)(z^2+36)}$ верно

$= \pi \left(\frac{4i e^{-20}}{8i \cdot 20} + \frac{6i e^{-30}}{-20 \cdot 12i} \right) = \frac{\pi}{40} \left(\frac{1}{e^{20}} - \frac{1}{e^{30}} \right) \leftarrow$ Ответ

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$$w = \frac{az+b}{cz+d}$$

z	w
1	1
i	0
$-i$	-1

$|z|<1$

$\operatorname{Im} w < 0$

$$\begin{cases} 1 = \dfrac{a+b}{ci+d} & c+d=a+b \\ 0 = \dfrac{ai+b}{ci+d} & ai=-b \\ -1 = \dfrac{-ai+b}{-ic+d} & \end{cases}$$

да

$$\begin{cases} a - ai = c+d \\ -2ai = ci - d \end{cases} \Rightarrow a(1-3i) = c(1+i)$$

$$c = \frac{a(1-3i)(1-i)}{2} = \frac{a(1-3-4i)}{2} = -a(1+2i)$$ да

$$d = a+b-c = a - ai + a(1+2i) = a(2+i)$$ да

$$w = \frac{az - ai}{-a(1+2i)z + a(z+i)} = \frac{z-i}{-(1+2i)z+z+i}$$

~~ответ~~ верно

4)
$$f(z) = \frac{z+2}{z^2+2z-8} = \frac{z+2}{(z+4)(z-2)} = \frac{z+2}{6}\left(\frac{1}{z-2} - \frac{1}{z+4}\right)$$

$$\frac{1}{z-2} = \frac{1}{z}\,\frac{1}{1-\frac{2}{z}} = \frac{1}{z}\sum_{n=0}^{\infty}\frac{2^n}{z^n} = \sum_{n=0}^{\infty}\frac{2^n}{z^{n+1}}$$

$$\frac{1}{z+4} = \frac{1}{4}\,\frac{1}{1-\left(-\frac{z}{4}\right)} = \frac{1}{4}\sum_{n=0}^{\infty}(-1)^n\frac{z^n}{4^n} = \sum_{n=0}^{\infty}(-1)^n\frac{z^n}{4^{n+1}}$$

можно и так

$$\Leftrightarrow \frac{1}{6}\sum_{n=0}^{\infty}\frac{2^n}{z^n} + \frac{1}{6}\sum_{n=0}^{\infty}\frac{2^{n+1}}{z^{n+1}} - \frac{1}{6}\sum_{n=0}^{\infty}(-1)^n\frac{z^{n+1}}{4^{n+1}}$$

$$-\frac{1}{3}\sum_{n=0}^{\infty}\frac{z^n(-1)^n}{4^{n+1}} = \frac{1}{3}\sum_{n=-\infty} 2^n z^n + \frac{1}{6} - \frac{1}{12} - \left(\frac{1}{6}-\frac{1}{12}\right)\sum_{n=1}^{\infty}(-1)^{n+1}\frac{z^n}{4^n}$$

$$= \frac{1}{3}\sum_{n=-\infty}^{-1} 2^n z^n + \frac{1}{12} + \sum_{n=1}^{\infty} (-1)^{n+1} \frac{z^n}{4^n} =$$

$$= \frac{1}{3}\sum_{n=-\infty}^{-1} 2^n z^n + \frac{1}{12} + \frac{1}{3}\sum_{n=1}^{\infty} (-1)^{n+1} \frac{z^n}{4^{n+1}}$$